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PHYSICAL PROPERTIES OF CRYSTALS

Static Photoelasticity of Gallium Phosphide Crystals

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Abstract—The piezo-optic effect (POE) in cubic GaP crystals (symmetry class $\bar{4}3m$) is studied in detail by interferometry. The relations for determining the absolute piezo-optic coefficients (POCs) π_{im} or their combinations on a sample of $X/45°$ cut at all allowable geometries of the experiment are recorded. The determination of a specific coefficient π_{im} at different experimental geometries on samples of right cuts and a $X/45°$ cut made it possible to find the π_{im} values with a high accuracy and reliability.

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INTRODUCTION

Gallium phosphide (GaP) crystals are characterized by a wide range of transparency (from 0.6 to 16 μm [1]) and can be used for electro-, piezo-, and acousto-optic light modulation in the IR region. In [2], only the difference in the absolute piezo-optic coefficients (POCs), $\pi_{11} - \pi_{12}$, and the coefficient π_{44} were determined from Raman scattering. In this study all absolute POCs were found independently. To this end we performed a complete analysis of the piezo-optic effect (POE) in gallium phosphide crystals. Specifically, the analytical description of the POE was carried out, the control stresses were measured by the interferometric method, and the absolute POCs were calculated both for the main geometries of the experiment and for the geometries that confirm the reliability of POC values. The changes in the optical path length per sample unit length and unit stress were determined; these changes characterize the modulating properties of the material.

RELATIONS FOR DETERMINING THE POCS OF GAP CRYSTALS

Gallium phosphide crystals belong to the $\bar{4}3m$ symmetry class; its POC matrix contains 12 coefficients π_{im} (the subscripts i and m indicate, respectively, the direction of the vector of light-wave oscillations and the direction in which uniaxial pressure acts in the crystallophysical coordinate system), among which only three POCs are independent: π_{11}, π_{12}, and π_{44}. The principal coefficients π_{im} (i, m = 1, 2, 3) are determined based on the known relation (see, for example, [3]):

$$\delta\Delta_k = -\frac{1}{2}\pi_{im}\sigma_m n_i^3 d_k + S_{km}\sigma_m d_k(n_i - 1), \tag{1}$$

where $\delta\Delta_k$ is the change in the optical path of light in a single-pass (for example, Mach–Zehnder) interferometer induced by mechanical stress σ_m, n_i is the refractive index of the sample, d_k is its thickness (the subscript k indicates the light propagation directions in the sample), and S_{km} are the elastic compliance constants of the crystal.

For the method of half-wave stress, where $\delta\Delta_k = \lambda/2$ ($\lambda = 632.8$ nm is the light wavelength) and $\sigma_m = \sigma_{im}$ is the half-wave stress, relation (1) yields

$$\pi_{im} = -\frac{\lambda}{n_i^3\sigma_{im}^o} + 2S_{km}\frac{n_i - 1}{n_i^3}, \tag{2}$$

where $\sigma_{im}^o = \sigma_{im}d_k$ is the control stress.

Relation (2) describes the POE for a right-cut sample (the faces of which are perpendicular to the optical-indicatrix axes X, Y, and Z). For example, the conditions of the experiment with $m = 1$ and $k = 3$ allow for two directions of the light oscillation (polarization) vector: $i = 1$ and 2. Then, based on (2), we obtain

$$\begin{aligned}\pi_{11} &= -\frac{\lambda}{n_1^3\sigma_{11}^o} + 2S_{21}\frac{n_1 - 1}{n_1^3},\\ \pi_{21} &= -\frac{\lambda}{n_1^3\sigma_{21}^o} + 2S_{21}\frac{n_1 - 1}{n_1^3}.\end{aligned} \tag{3}$$

Here, $S_{21} = S_{12}$ and $\pi_{21} = \pi_{12}$, in correspondence with the matrices of elastic coefficients and POCs of the crystals belonging to the symmetry class $\bar{4}3m$ [1, 4].

Note that (2) is valid for samples with parallel optical faces. In the case of insignificant nonparallellity in real samples (the angle between the faces $\alpha = (2\text{-}5) \times 10^{-2}$ deg), relation (2) takes the form [3, 5]

$$\pi_{im} = -\frac{\lambda}{2n_i^3}\left(\frac{1}{\sigma_{im}^o} + \frac{1}{\sigma_{im}'^o}\right) + 2S_{km}\frac{n_i - 1}{n_i^3}, \qquad (4)$$

where $\sigma_{im}'^{10}$ is the control stress of the sample rotated by 180° along the direction of uniaxial pressure; i.e., the lower and upper faces (on which the force F_m acts) are interchanged.

Let us write relations similar to (4) for the coefficient π_{44}. To this end, we will use Eq. [6], which describes the change in the optical path in the sample of $X/45°$ cut (figure) for the experimental conditions $i = 4$ (or $\bar{4}$), $m = 4$ ($\bar{4}$), and $k = \bar{4}$ (4):

$$\begin{aligned}\delta\Delta_{\bar{4}(4)} &= -\frac{\pi_{11} + \pi_{12} + \pi_{44}}{4}\sigma d_{\bar{4}(4)} n_1^3 \\ &+ \frac{2S_{11} + 2S_{12} - S_{44}}{4}\sigma d_{\bar{4}(4)}(n_1 - 1),\end{aligned} \qquad (5)$$

where σ is the mechanical stress (uniaxial pressure acting along the 4 or $\bar{4}$ direction (figure)).

Note that relation (5) contains in fact two independent equations for the symmetric conditions of the experiment, which are indicated above (in parentheses and without them).

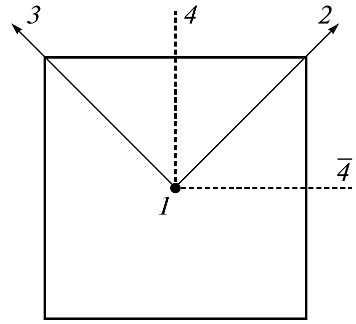
The other two equations for determining π_{44} correspond to the experimental conditions $i = 4$ (or $\bar{4}$), $m = 4$, and $k = 1$:

$$\delta\Delta_1 = -\frac{\pi_{11} + \pi_{12} \pm \pi_{44}}{4}\sigma d_1 n_1^3 + S_{12}\sigma d_1(n_1 - 1). \qquad (6)$$

A similar relation will be obtained under symmetric conditions: $i = \bar{4}$ (4), $m = \bar{4}$, and $k = 1$.

The superscript and subscript at the coefficient π_{44} in relation (6) correspond to the conditions $i = 4$ and $\bar{4}$, respectively. Since there is no criterion for unambiguously determining the positive directions of the X, Y, and Z axes and, correspondingly, the directions 4 and $\bar{4}$ in cubic crystals (see [6] for details), relation (6) cannot be used to unambiguously determine the sign of the coefficient π_{44}. Relations (5) are free of this contradiction; therefore, they make it possible to determine both the values and the sign of the POC π_{44}. However, the absolute value of π_{44} can be determined more exactly from (6), because the error of the elastic contribution (the second term) to $\delta\Delta_1$ is small (it is determined by only the error of one elastic compliance coefficient (S_{12}). Therefore, it is necessary to determine the sign of π_{44} from (5) and its magnitude from (6).

Let us reduce (5) and (6) to the form corresponding to the method of half-wave (control) stresses. To this end we will substitute $\lambda/2$ instead of $\delta\Delta_k$ and the half-wave stress σ_{im} instead of σ into these relations and take into account that $\sigma_{im} d_k = \sigma_{im}^o$. In addition, to eliminate the error of small nonparallelity of the sample optical faces, we will replace $1/\sigma_{im}^o$ by the averaged value of inverse control stresses $(1/\sigma_{im}^o + 1/\sigma_{im}'^o)/2$ (by analogy with (4)). As a result, (5) yields the following relation for the conditions $i = m = 4$and $k = \bar{4}$:



Schematic diagram of the sample of $X/45°$ cut: directions *1, 2*, and *3* correspond to the crystallophysical axes X, Y, and Z.

$$\begin{aligned}\pi_{11} + \pi_{12} + \pi_{44} &= -\frac{\lambda}{n_1^3}\left(\frac{1}{\sigma_{44}^o} + \frac{1}{\sigma_{44}'^o}\right) \\ &+ (2S_{11} + 2S_{12} - S_{44})\frac{n_1 - 1}{n_1^3}.\end{aligned} \qquad (7)$$

For the symmetric conditions $i = m = \bar{4}$ and $k = 4$, we have another independent relation:

$$\begin{aligned}\pi_{11} + \pi_{12} + \pi_{44} &= -\frac{\lambda}{n_1^3}\left(\frac{1}{\sigma_{\bar{4}\bar{4}}^o} + \frac{1}{\sigma_{\bar{4}\bar{4}}'^o}\right) \\ &+ (2S_{11} + 2S_{12} - S_{44})\frac{n_1 - 1}{n_1^3}.\end{aligned} \qquad (8)$$

Based on (6), we can write the following relations for the conditions $i = 4$, $m = 4$, $k = 1$ and $i = \bar{4}$, $m = 4$, $k = 1$:

$$\pi_{11} + \pi_{12} + \pi_{44} = -\frac{\lambda}{n_1^3}\left(\frac{1}{\sigma_{44}^o} + \frac{1}{\sigma_{44}'^o}\right) + 4S_{12}\frac{n_1 - 1}{n_1^3}. \qquad (9)$$

$$\pi_{11} + \pi_{12} - \pi_{44} = -\frac{\lambda}{n_1^3}\left(\frac{1}{\sigma_{\bar{4}4}^o} + \frac{1}{\sigma_{\bar{4}4}'^o}\right) + 4S_{12}\frac{n_1 - 1}{n_1^3}. \qquad (10)$$

Having subtracted (10) from (9), we can write a simple relation for determining π_{44}:

$$\pi_{44} = -\frac{\lambda}{2n_1^3}\left(\frac{1}{\sigma_{44}^o} + \frac{1}{\sigma_{44}'^o} - \frac{1}{\sigma_{\bar{4}4}^o} - \frac{1}{\sigma_{\bar{4}4}'^o}\right). \qquad (11)$$

Since (11) does not contain the principal POCs π_{11} and π_{12} or the elastic contribution, the error in determining π_{44} is small.

RELATIONS FOR CONFIRMING THE RELIABILITY OF THE POE STUDY

The data of different researchers on the physical properties of GaP crystals differ significantly [1]; therefore, it is important to confirm the reliability of the POE study. To this end, we will determine the principal POCs on the sample of $X/45°$cut (figure) and compare them with the principal POCs obtained on the right-cut sample, the POE in which is described by simple relations (1)–(4).

Let us sum relations (9) and (10). As a result, we have

$$\pi_{11} + \pi_{12} = -\frac{\lambda}{2n_1^3}\left(\frac{1}{\sigma_{44}^o} + \frac{1}{\sigma_{44}^{\prime o}} + \frac{1}{\sigma_{\bar{4}4}^o} + \frac{1}{\sigma_{\bar{4}4}^{\prime o}}\right) + 4S_{12}\frac{n_1 - 1}{n_1^3}. \tag{12}$$

The value of this sum, $\pi_{11} + \pi_{12}$, can be compared with the sum of independent POCs π_{11} and π_{12} derived from (2)–(4).

Note that formulas (5) and (6) are written based on the method of cutting the optical indicatrix perturbed by mechanical stress and the characteristic surface of the strain tensor by straight lines coinciding, respectively, with the direction of the vector **i** of light wave oscillations and the light propagation direction (the method is described in [6]). Using this method, we can write the other relations for determining the principal POCs π_{im} (i, m = 1, 2, 3) or their sums for the sample of $X/45°$ cut. For example, for the condition $i = m = 1$ and $k = \bar{4}$ (or 4), we have

$$\delta\Delta_{\bar{4}(4)} = -\frac{1}{2}\pi_{11}d_{\bar{4}(4)}\sigma n_1^3 + S_{12}d_{\bar{4}(4)}\sigma(n_1 - 1). \tag{13}$$

Relation (13) for the half-wave stresses method, with allowance for the nonparallelity of optical faces of real samples, is transformed as follows:

$$\pi_{11} = -\frac{\lambda}{2n_1^3}\left(\frac{1}{\sigma_{11}^o} + \frac{1}{\sigma_{11}^{\prime o}}\right) + 2S_{12}\frac{n_1 - 1}{n_1^3}; \tag{14}$$

when changing the light polarization from $i = 1$ to $i = 4$ (or $i = \bar{4}$ if $k = 4$) for the previous conditions of the experiment, we obtain the relations for determining the POC π_{12}:

$$\delta\Delta_{\bar{4}(4)} = -\frac{1}{2}\pi_{12}d_{\bar{4}(4)}\sigma n_1^3 + S_{12}d_{\bar{4}(4)}\sigma(n_1 - 1) \tag{15}$$

or

$$\pi_{12} = -\frac{\lambda}{2n_1^3}\left(\frac{1}{\sigma_{41}^o} + \frac{1}{\sigma_{41}^{\prime o}}\right) + 2S_{12}\frac{n_1 - 1}{n_1^3} \tag{16}$$

and a similar relation for π_{12}, with σ_{14}^o and $\sigma_{14}^{\prime o}$ replaced by $\sigma_{\bar{4}1}^o$ and $\sigma_{\bar{4}1}^{\prime o}$.

Let us change the light polarization for the conditions given by formulas (7) and (8): from $i = 4$ (or $\bar{4}$) to $i = 1$; as a result, we have

$$\delta\Delta_{4(\bar{4})} = -\frac{1}{2}\pi_{12}\sigma d_{4(\bar{4})}n_1^3 + \frac{1}{4}(2S_{11} + 2S_{12} - S_{44})\sigma d_{4(\bar{4})}(n_1 - 1) \tag{17}$$

or

$$\pi_{12} = -\frac{\lambda}{2n_1^3}\left(\frac{1}{\sigma_{14}^o} + \frac{1}{\sigma_{14}^{\prime o}}\right) + \frac{1}{2}(2S_{11} + 2S_{12} - S_{44})\frac{n_1 - 1}{n_1^3}. \tag{18}$$

A similar relation for π_{12} can be obtained by replacing σ_{14}^o and $\sigma_{14}^{\prime o}$ by $\sigma_{1\bar{4}}^o$ and $\sigma_{1\bar{4}}^{\prime o}$.

Relations (11) and (12) are derived from (6) for the experimental conditions $i = 4(\bar{4})$, $m = 4$, and $k = 1$; having changed these conditions by the symmetric ones ($i = \bar{4}$ (or 4), $m = \bar{4}$, and $k = 1$), we obtain two more independent relations for determining π_{44} and the sum $\pi_{11} + \pi_{12}$:

$$\pi_{44} = -\frac{\lambda}{2n_1^3}\left(\frac{1}{\sigma_{\bar{4}4}^o} + \frac{1}{\sigma_{\bar{4}4}^{\prime o}} - \frac{1}{\sigma_{4\bar{4}}^o} - \frac{1}{\sigma_{4\bar{4}}^{\prime o}}\right), \tag{19}$$

$$\pi_{11} + \pi_{12} = -\frac{\lambda}{2n_1^3}\left(\frac{1}{\sigma_{\bar{4}\bar{4}}^o} + \frac{1}{\sigma_{\bar{4}\bar{4}}^{\prime o}} + \frac{1}{\sigma_{4\bar{4}}^o} + \frac{1}{\sigma_{4\bar{4}}^{\prime o}}\right) + 4S_{12}\frac{n_1 - 1}{n_1^3}. \tag{20}$$

Relations (12), (14), (16), and (18)–(20) were used to find the POCs π_{11}, π_{12}, $\pi_{11} + \pi_{12}$, and π_{44} based on the additional geometries of the experiment on the sample of $X/45°$ cut.

RESULTS AND DISCUSSION

A study was performed on samples in the form of cubes ~8 × 8 × 8 mm in size. The experimental values of the control stresses σ_{im}^o and $\sigma_{im}^{\prime o}$, measured using a system based on a Mach–Zehnder interferometer, as well as the values of the POCs π_{11}, π_{12}, $\pi_{11} + \pi_{12}$, and π_{44} obtained from different geometries of the experiment, are listed in Table 1. To calculate the coefficients π_{im}, we used the values of the refractive index of GaP crystals, $n_1 = 3.313$ [1] (for $t = 20°C$, $\lambda = 0.63$ μm), and the elastic coefficients $S_{11} = 9.73$, $S_{12} = -2.99$, and $S_{44} = 14.19$ [7, 8] (in 10^{-12} m^2/H = 1 Br (Brewster) units).

We should note the following. The values of the POCs π_{im} are given with their errors in Table 1. These errors are calculated as the rms values of errors of the first and second terms, which enter the equations for determining π_{im}. Let us calculate, for example, the

Table 1. Control stresses $\sigma^o_{im} = \sigma_{im} d_k$ and absolute POCs π_{im} of GaP crystals

Number of experimental geometries	Experimental conditions			σ^o_{im} and σ'^o_{im}, kG/cm		π_{im}, Br
	m	k	i			
Samples of right cuts						
1	1	3	1	$\sigma^o_{11} = -17.6$	$\sigma'^o_{11} = -16.1$	$\pi_{11} = -1.43 \pm 0.11$
2			2	$\sigma^o_{21} = +81$	$\sigma'^o_{21} = +115$	$\pi_{12} = -0.19 \pm 0.03$
3	1	2	1	$\sigma^o_{11} = -19.5$	$\sigma'^o_{11} = -14.5$	$\pi_{11} = -1.45 \pm 0.11$
4			3	$\sigma^o_{31} = +61$	$\sigma'^o_{31} = +220$	$\pi_{12} = -0.19 \pm 0.03$
5	2	1	2	$\sigma^o_{22} = -16.5$	$\sigma'^o_{22} = -16.8$	$\pi_{11} = -1.45 \pm 0.11$
6			3	$\sigma^o_{32} = +110$	$\sigma'^o_{32} = +85$	$\pi_{12} = -0.20 \pm 0.03$
7	2	3	2	$\sigma^o_{22} = -17.1$	$\sigma'^o_{22} = -16.4$	$\pi_{11} = -1.44 \pm 0.11$
8			1	$\sigma^o_{12} = +95$	$\sigma'^o_{12} = +93$	$\pi_{12} = -0.19 \pm 0.03$
Sample of X/45° cut						
9	1	4	1	$\sigma^o_{11} = -16$	$\sigma'^o_{11} = -17$	$\pi_{11} = -1.46 \pm 0.11$
10			$\bar{4}$	$\sigma^o_{\bar{4}1} = +110$	$\sigma'^o_{\bar{4}1} = +83$	$\pi_{12} = -0.19 \pm 0.03$
11	1	$\bar{4}$	1	$\sigma^o_{11} = -16.4$	$\sigma'^o_{11} = -17.5$	$\pi_{11} = -1.43 \pm 0.11$
12			4	$\sigma^o_{41} = +120$	$\sigma'^o_{41} = +80$	$\pi_{12} = -0.20 \pm 0.03$
13	4	$\bar{4}$	4	$\sigma^o_{44} = -14.7$	$\sigma'^o_{44} = -22.5$	$\pi_{44} = -1.60 \pm 0.25$
14			1	$\sigma^o_{14} = +90$	$\sigma'^o_{14} = +27.5$	$\pi_{12} = -0.19 \pm 0.03$
15	4	1	4	$\sigma^o_{44} = -16$	$\sigma'^o_{44} = -15.3$	$\pi_{44} = -1.38 \pm 0.12$
16			$\bar{4}$	$\sigma^o_{\bar{4}4} = +70$	$\sigma'^o_{\bar{4}4} = +77$	$\pi_{11} + \pi_{12} = -1.6 \pm 0.12$
17	$\bar{4}$	4	$\bar{4}$	$\sigma^o_{\bar{4}\bar{4}} = -18.0$	$\sigma'^o_{\bar{4}\bar{4}} = -19$	$\pi_{44} = -1.52 \pm 0.25$
18			1	$\sigma^o_{1\bar{4}} = +45$	$\sigma'^o_{1\bar{4}} = +38$	$\pi_{12} = -0.19 \pm 0.03$
19	$\bar{4}$	1	$\bar{4}$	$\sigma^o_{\bar{4}\bar{4}} = -17.5$	$\sigma'^o_{\bar{4}\bar{4}} = -15.6$	$\pi_{44} = -1.34 \pm 0.11$
20			4	$\sigma^o_{4\bar{4}} = +55$	$\sigma'^o_{4\bar{4}} = +90$	$\pi_{11} + \pi_{12} = -1.58 \pm 0.12$

coefficient π_{11} (in Br units) for the initial data (Table 1, row 1) by substituting them into (4):

$$\pi_{11} = -\frac{632.8}{2(3.313)^3}\left(\frac{-1}{-17.6} + \frac{-1}{-16.1}\right) + 2\frac{-2.99}{(3.313)^3}(3.313 - 1) \quad (21)$$
$$= -1.05 \pm 0.105 \;-0.38 \pm 0.019 \;= -1.43 \pm 0.11.$$

Note that the minus sign in the numerators of (21) was taken from Table 1, where it precedes σ^o_{im} and σ'^o_{im} and means that the compressive stress reduces the optical path in the sample (which is determined by rotating a plane-parallel plate placed in the way of an interferometric ray passing through the sample), whereas the minus sign in the denominators of (21) corresponds to the sign of compressive mechanical stress.

The error of the coefficient π_{11} is calculated proceeding from the error of ±10% for the sum $1/\sigma^o_{11}$ + $1/\sigma'^o_{11}$ (this is an objective error, which is verified by multiple measurements of σ^o_{im} and σ'^o_{im} on different samples) and an error of ±5% for S_{12}. We should emphasize that the elastic coefficient S_{12} in [7] was determined with an error of ±1.8%; however, since the static experiment is characterized by nonuniform stress distribution over the sample, the use of the coefficients S_{12} with an error of ±5% is quite justified.

Note that, when calculating the POC π_{44} and the sum $\pi_{11} + \pi_{12}$ (relations (11), (12)), the error of the expression in the parentheses was determined as the rms value of the ±10% errors for the sums $1/\sigma^o_{44} + 1/\sigma'^o_{44}$ and $1/\sigma^o_{\bar{4}4} + 1/\sigma'^o_{\bar{4}4}$ (this also concerns relations (19) and (20)). When determining π_{44} based on complex relations (7) and (8), it is also necessary to take into account the rms errors of the sums of the principal POCs π_{11} and π_{12} and all elastic compliance coefficients S_{km}.

The POCs π_{11} obtained from the four geometries of the experiment on the sample of direct cuts (Table 1, rows 1, 3, 5, 7) and from the two geometries of the experiment on the sample of X/45° cut (rows 9, 11) coincide within the experimental error. This is also true for the coefficient π_{12} (rows 2, 4, 6, 8, 10, 12). Furthermore, we will use the arithmetic means of the POCs π_{11} and π_{12} obtained from the aforementioned experimental geometries (see below).

It is remarkable that relations (14) and (16) for π_{11} and π_{12} obtained for the sample of X/45° cut are identical to the corresponding relations for the samples of right cuts (they can easily be written based on (4)). For example, for the conditions $i = 1$, $m = 2$, and $k = 3$, we have

$$\pi_{12} = -\frac{\lambda}{2n_1^3}\left(\frac{1}{\sigma^o_{12}} + \frac{1}{\sigma'^o_{12}}\right) + 2S_{32}\frac{n_1 - 1}{n_1^3}. \tag{22}$$

A comparison of relations (22) and (16) suggests that the sums $1/\sigma^o_{12} + 1/\sigma'^o_{12}$ and $1/\sigma^o_{41} + 1/\sigma'^o_{41}$ are equal. These sums are also equal to similar sums based on the control stresses σ^o_{21}, σ^o_{32}, σ^o_{31}, and $\sigma^o_{\bar{4}1}$ (rows 2, 4, 6, 8, 10, 12), since the following equalities are valid for cubic crystals: $\sigma^o_{12} = \sigma^o_{21} = \sigma^o_{32} = \sigma^o_{31}$ and $\sigma^o_{41} = \sigma'^o_{41}$ (in correspondence with the note after (16)). Obviously, sums $1/\sigma^o_{11} + 1/\sigma'^o_{11}$ and $1/\sigma^o_{22} + 1/\sigma'^o_{22}$ for the samples of right cuts and the sample of X/45° cut are also equal within the error of the experiment (rows 1, 3, 5, 7, 9, 11).

The sum $\pi_{11} + \pi_{12} = -1.61 \pm 0.12$, which was determined on the sample of X/45° cut (rows 16, 20), coincides with a high accuracy with the sum of independent POCs π_{11} and π_{12} (determined on the sample of right cuts), which is an additional justification for the π_{11} and π_{12} values.

Note significant differences in the control stresses σ^o_{im} and σ'^o_{im}, especially in the cases where they are large (for example, Table 1, rows 4, 12, 14, 20). However, the sums $(1/\sigma^o_{im} + 1/\sigma'^o_{im})$ for symmetrically identical geometries of the experiment are equal within the errors (±10%) in determining these sums (for example, pairwise rows 1 and 3, 2 and 4, 14 and 18, 16 and 20). This confirms the efficiency of the technique [3, 5] for taking into account the influence of the small nonparallelity of optical faces of real samples on the error in determining POCs.

The coefficient π_{44} was found from four geometries of the experiment (rows 13, 15, 17, 19). The small error in determining π_{44} under the experimental conditions $m = 4$, $k = 1$, and $i = 4(\bar{4})$ and symmetric conditions $m = \bar{4}$, $k = 1$, and $i = \bar{4}$ (4) is due to the fact that relations (11) and (19), which correspond to these conditions, do not contain the principal POCs and elastic coefficients S_{km}.

At the same time, under the conditions $m = 4$, $k = \bar{4}$, and $i = 4$ and the symmetric (with respect to them) conditions $m = \bar{4}$, $k = 4$, and $i = \bar{4}$ (rows 13, 17), the coefficient π_{44} must be determined based on relations (7) and (8). Since the latter contain the sum of the principal POCs π_{11} and π_{12}, as well as the complex sum of the coefficients S_{km}, the error of π_{44} is much larger. In addition, having substituted the values of the coefficients S_{km} from [7] into (7) and (8), we obtain $\pi_{44} = -0.41 \pm 0.25$ and $\pi_{44} = -0.33 \pm 0.24$ (in Br units). These large errors and the inconsistency between π_{44} and the values found based on simple relations (11) and (19) (rows 15, 19) can be explained by only the nonobjective value of the sum S_{km}, which enters (7) and (8). In this study this sum S_{km} was found based on relations (18) in order to determine the POCs π_{12} on the sample of X/45° cut, because the coefficient π_{12}, as well as π_{11}, is found from six geometries of the experiment with a high accuracy (the absolute error in determining π_{12} is very small: ±0.03 Br (Table 1, rows 2, 4, 6, 8, 10, 12)). The thus determined ΣS_{km} value is –19.4 ± 1.6 (in Br). Having substituted this ΣS_{km} value into (7) and (8), we obtain, respectively, $\pi_{44} = -1.60 \pm 0.25$ and $\pi_{44} = -1.52 \pm 0.25$ (in Br); the calculations were performed using the arithmetic means of π_{11} and π_{12} from Table 1. These values of the coefficient π_{44} correspond (within the error of their determination to the π_{44} values derived based on relations (11) and (19) (Table 1, rows 15, 19). Furthermore, we will use the arithmetic means π_{44} from rows 15 and 19 of Table 1, because they are determined with a small error (±0.11 Br), as well as the arithmetic means of the coefficients π_{11} and π_{12}, each of which is determined from six experimental geometries (in Br):

$$\pi_{11} = -1.44 \pm 0.11; \quad \pi_{12} = -0.19 \pm 0.03; \quad \pi_{44} = -1.36 \pm 0.12. \tag{23}$$

A comparison with the data in the literature. Canal et al. [2] determined the POC difference $\pi_{11} - \pi_{12}$ and the coefficient π_{44} (in Br) by Raman spectroscopy: $\pi_{11} - \pi_{12} = -0.97$ and $\pi_{44} = -1.16$. According to the values reported in (23), both the difference $\pi_{11} - \pi_{12} =$

Table 2. Piezo-optic $\delta\Delta_k(\pi_{im})$ and elastic $\delta\Delta_k(S_{km})$ contributions to the induced change in the optical path length $\delta\Delta_k/(\sigma_m d_k)$

Number of experimental geometries	Experimental conditions			$\frac{\delta\Delta_k}{\sigma_m d_k}$, Br	$\frac{\delta\Delta_k(\pi_{im})}{\delta\Delta_k}$, %	$\frac{\delta\Delta_k(S_{km})}{\delta\Delta_k}$, %
	m	k	i			
1	1	3	1	+19.3	+136	−36
2	1	3	2	−3.4	+100	−200
3	4	$\bar{4}$	4	+16.0	+170	−70
4	4	1	4	+20.3	+134	−34
5	4	1	$\bar{4}$	−4.5	+54	−154

−1.25 Br and the coefficient π_{44} significantly exceed the POC values obtained in [2] (by 29 and 17%, respectively). These differences can be due to two factors: the large error in determining POCs in [2] (unfortunately, the errors in studying the POE were not reported in [2]) and the dependence of the POC value on the technology of GaP crystal growth. The latter reason appears to be weightier, because many physical properties of gallium phosphide crystals depend strongly on their growth conditions [1].

Based on the coefficients π_{im} and S_{km}, we can find the change in the optical path length $\delta\Delta_k$ per unit sample length and unit mechanical stress $\delta\Delta_k/(\sigma_m d_k)$ and compare the piezo-optic $\delta\Delta_k(\pi_{im})$ and elastic $\delta\Delta_k(S_{km})$ contributions to $\delta\Delta_k$. To this end, we will write the following relation based on (1):

$$\frac{\delta\Delta_k}{\sigma_m d_k} = -\frac{1}{2}\pi_{im}n_i^3 + S_{km}(n_i - 1). \qquad (24)$$

This relation is valid for the principal POCs π_{im} (i, m = 1, 2, 3). For the other geometries of the experiment, the analogs of (24) can easily be written based on relations (5) and (6). Table 2 contains the values $\delta\Delta_k/(\sigma_m d_k)$ and the corresponding contributions (piezo-optic and elastic) to the change in the optical path length for five experimental geometries. The geometry with large $\delta\Delta_k/(\sigma_m d_k)$ values is interesting for practical applications (for example, photoelastic light modulation [9–12]). Table 2 shows that most of the experimental geometries (rows 1, 3, and 4 in Table 2 and symmetrically identical geometries) are characterized by large induced changes in the optical path length. For comparison, the maximum $\delta\Delta_k/(\sigma_m d_k)$ values for lithium niobate crystals, both pure and doped by magnesium oxide, are 8.0–13.7 Br [3], whereas for langasite crystals the corresponding values are 6.25–6.65 Br [5]. The main contribution to $\delta\Delta_k$ for the aforementioned geometries of the experiment is from the POE (larger than the elasticity contribution by a factor of 2.5–4.0). The reason is that the piezo-optic contribution is formed by the large coefficient π_{11} (Table 2, row 1) or the large value of the POC sum $\pi_{11} + \pi_{12} + \pi_{44} = -2.99$ Br (Table 2, rows 3, 4). The small $\delta\Delta_k/(\sigma_m d_k)$ values correspond to the small POC π_{12} (Table 2, row 2) or the small POC combination $\pi_{11} + \pi_{12} - \pi_{44} = -0.27$ Br. Thus, GaP crystals should be considered among the best photoelastic materials, as follows from both the values of POCs π_{11} and π_{44} and the large $\delta\Delta_k/(\sigma_m d_k)$ values.

CONCLUSIONS

The efficiency of the technique for eliminating the errors of the POE study, which are caused by the small real nonparallelity of the sample faces oriented perpendicularly to the optical ray direction, was confirmed by the example of GaP crystals. The relations for determining the absolute POCs π_{im} or their combinations on the sample of X/45° cut in all possible geometries of the experiment are presented. It is shown that the relations for determining the POCs π_{11} and π_{12} on the sample of X/45° cut are identical to the similar relations for the sample of right cuts. The correspondence of the POCs π_{im} or their combinations obtained on the aforementioned samples confirms the reliability of their values.

Based on the POCs π_{im}, the specific (per unit mechanical stress and sample unit length) changes in the optical path length $\delta\Delta_k/(\sigma_m d_k)$, which characterize the modulation properties of the material, were determined. Concerning the POC and $\delta\Delta_k/(\sigma_m d_k)$ values, gallium phosphide crystals should also be considered among the best photoelastic materials.

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